MATH 3060 Tutorial 1

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September 21, 2022

1. True or false:

   (a) A 2π-periodic continuous function on $\mathbb{R}$ is uniformly continuous. **T**

   (b) Any continuous function $f : [0,1] \to \mathbb{R}$ is Riemann integrable on $[0,1]$. **T**

   (c) Let $f, g : [0,1] \to \mathbb{R}$ be Riemann integrable functions on $[0,1]$, then the product fg is also Riemann integrable on $[0,1]$. **T**

   (d) Let $f : [0,1] \to \mathbb{R}$ with $f(0) = 0$, and

   $$f(x) = \frac{1}{\sqrt{x}}$$

   for $x \neq 0$. Then f is Riemann integrable on $[0,1]$ with $\int_0^1 f = 2$, but f^2 is not Riemann integrable on $[0,1]$. **F**

2. Find the Fourier coefficients a_n, b_n, c_n of the 2π periodic function f with

   $$f(x) = \sin\frac{x}{2}$$

   for $x \in (-\pi, \pi]$.

3. (a) Let f_1 be the 1-periodic extension of the function $x - \frac{1}{2}$ on $[0,1]$, show that

   $$f_1(x) \sim -\frac{1}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\sin(2n\pi x)$$

   .

   (b) Let f_2 be the 1-periodic extension of the function $\frac{x^2}{2} - \frac{x}{2} + \frac{1}{12}$ on $[0,1]$, show that

   $$f(x) \sim \frac{1}{2\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2}\cos(2n\pi x)$$

   .

   (c) Can you find a 1-periodic function f with

   $$f(x) \sim \sum_{n=1}^{\infty}\frac{1}{n^4}\cos(2n\pi x)$$

4. Let f, g be 1-periodic functions integrable on $[0,1]$ with $\int_0^1 g(t)dt = 0$. Imitate the proof of the Riemann Lebesgue lemma, show that

$$\lim_{n\to\infty} \int_0^1 f(t)g(nt)dt = 0$$